NAG Toolbox for MATLAB

g05pa

1 Purpose

g05pa generates a realization of a univariate time series from an autoregressive moving average (ARMA) model. The realization may be continued or a new realization generated at subsequent calls to g05pa.

2 Syntax

```
[var, x, iseed, r, ifail] = g05pa(mode, xmean, phi, theta, avar, n,
igen, iseed, r, 'ip', ip, 'iq', iq)
```

3 Description

Let the vector x_t, denote a time series which is assumed to follow an autoregressive moving average (ARMA) model of the form:

$$\begin{aligned} x_t - \mu = \quad & \phi_1(x_{t-1} - \mu) + \phi_2(x_{t-2} - \mu) + \cdots + \phi_p(x_{t-p} - \mu) + \\ & \epsilon_t - \theta_1\epsilon_{t-1} - \theta_2\epsilon_{t-2} - \cdots - \theta_q\epsilon_{t-q} \end{aligned} \tag{1}$$

where ϵ_t, is a residual series of independent random perturbations assumed to be Normally distributed with zero mean and variance σ^2. The parameters $\{\phi_i\}$, for $i = 1, 2, \ldots, p$, are called the autoregressive (AR) parameters, and $\{\theta_j\}$, for $j = 1, 2, \ldots, q$, the moving average (MA) parameters. The parameters in the model are thus the p ϕ values, the q θ values, the mean μ and the residual variance σ^2.

g05pa sets up a reference vector containing initial values corresponding to a stationary position using the method described in Tunnicliffe–Wilson 1979. The function can then return a realization of $x_1, x_2, \ldots, x_n$. On a successful exit, the recent history is updated and saved in the reference vector **r** so that g05pa may be called again to generate a realization of $x_{n+1}, x_{n+2}, \ldots$,etc. See the description of the parameter **mode** in Section 5 for details.

One of the initialization functions g05kb (for a repeatable sequence if computed sequentially) or g05kc (for a non-repeatable sequence) must be called prior to the first call to g05pa.

4 References

Knuth D E 1981 *The Art of Computer Programming (Volume 2)* (2nd Edition) Addison–Wesley

Tunnicliffe–Wilson G 1979 Some efficient computational procedures for high order ARMA models *J. Statist. Comput. Simulation* **8** 301–309

5 Parameters

5.1 Compulsory Input Parameters

1: **mode – int32 scalar**

A code for selecting the operation to be performed by the function:

mode $= 0$

Set up reference vector only.

mode $= 1$

Generate terms in the time series using reference vector set up in a prior call to g05pa.

mode $= 2$

Set up reference vector and generate terms in the time series.

Constraint: $0 \leq$ **mode** ≤ 2.

2: **xmean – double scalar**

The mean of the time series.

3: **phi**$(*)$ **– double array**

Note: the dimension of the array **phi** must be at least $\max(1, \mathbf{ip})$.

The autoregressive coefficients of the model, $\phi_1, \phi_2, \ldots, \phi_p$.

4: **theta**$(*)$ **– double array**

Note: the dimension of the array **theta** must be at least $\max(1, \mathbf{iq})$.

The moving average coefficients of the model, $\theta_1, \theta_2, \ldots, \theta_q$.

5: **avar – double scalar**

The variance of the normal perturbations, σ^2.

Constraint: **avar** ≥ 0.0.

6: **n – int32 scalar**

n, the number of observations to be generated.

Constraint: **n** ≥ 0.

7: **igen – int32 scalar**

Must contain the identification number for the generator to be used to return a pseudo-random number and should remain unchanged following initialization by a prior call to g05kb or g05kc.

8: **iseed**(4) **– int32 array**

Contains values which define the current state of the selected generator.

9: **r**$(\mathbf{nr})$ **– double array**

If **mode** $= 1$, the reference vector from the previous call to g05pa.

5.2 Optional Input Parameters

1: **ip – int32 scalar**

Default: The dimension of the array **phi**.

p, the number of autoregressive coefficients supplied.

Constraint: **ip** ≥ 0.

2: **iq – int32 scalar**

Default: The dimension of the array **theta**.

q, the number of moving average coefficients supplied.

Constraint: $\mathbf{iq} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

nr

5.4 Output Parameters

1: **var – double scalar**

The proportion of the variance of a term in the series that is due to the moving-average (error) terms in the model. The smaller this is, the nearer is the model to non-stationarity.

2: **x**$(*)$ **– double array**

Note: the dimension of the array **x** must be at least $\max(1, \mathbf{n})$.

Contains the next n observations from the time series.

3: **iseed**(4) **– int32 array**

Contains updated values defining the new state of the selected generator.

4: **r**$(\mathbf{nr})$ **– double array**

The reference vector.

5: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

If on entry **ifail** $= 0$ or -1, explanatory error messages are output on the current error message unit (as defined by x04aa).

Errors or warnings detected by the function:

ifail $= 1$

On entry, $\mathbf{n} < 0$.

ifail $= 2$

On entry, $\mathbf{ip} < 0$.

ifail $= 3$

On entry, $\mathbf{iq} < 0$.

ifail $= 4$

On entry, $\mathbf{avar} < 0$.

ifail $= 5$

On entry, $\mathbf{mode} < 0$,
or $\mathbf{mode} > 2$.

ifail $= 6$

On entry, $\mathbf{nr} < \mathbf{ip} + \mathbf{iq} + 5 + \max(\mathbf{ip}, \mathbf{iq} + 1)$.

ifail $= 7$

phi does not define a stationary autoregressive process.

ifail $= 8$

Either **r** has been corrupted or the value of **ip** or **iq** is not the same as when **r** was set up in a previous call with **mode** $= 0$ or 2.

7 Accuracy

The errors in the initialization process should be very much smaller than the error term; see Tunnicliffe–Wilson 1979.

8 Further Comments

The time taken by g05pa is essentially of order $(\mathbf{ip})^2$.

Note: g05kb and g05kc must be used with care if this function is used as well. The reference vector, as mentioned before, contains a copy of the recent history of the series. This will not be altered properly by calls to any of the above functions. A call to g05kb or g05kc should be followed by a call to g05pa with **mode** $= 0$ to re-initialize the time series reference vector in use. To maintain repeatability with g05kb, the calls to g05pa should be performed in the same order and at the same point or points in the simulation every time g05kb is used. When the generator state is saved and restored using the parameters **igen** and **iseed**, the time series reference vector must be saved and restored as well.

The ARMA model for a time series can also be written as:

$$(x_n - E) = A_1(x_{n-1} - E) + \cdots + A_{NA}(x_{n-NA} - E) + B_1 a_n + \cdots + B_{NB} a_{n-NB+1}$$

where

x_n is the observed value of the time series at time n,

NA is the number of autoregressive parameters, A_i,

NB is the number of moving average parameters, B_i,

E is the mean of the time series,

and

a_t is a series of independent random Standard Normal perturbations.

This is the form used in g05pa. This is related to the form given in Section 3 by:

$B_1^2 = \sigma^2$,

$B_{i+1} = -\theta_i \sigma = -\theta_i B_1, \qquad i = 1, 2, \ldots, q$,

$NB = q + 1$,

$E = c$,

$A_i = \phi_i, \qquad i = 1, 2, \ldots, p$,

$NA = p$.

9 Example

```
mode = int32(0);
xmean = 0;
phi = [0.4;
     0.2];
theta = [];
avar = 1;
```

```
n = int32(10);
igen = int32(1);
iseed = [int32(1762543);
     int32(9324783);
     int32(4234401);
     int32(742355)];
r = zeros(9, 1);
[var, x, iseedOut, rOut, ifail] = ...
    g05pa(mode, xmean, phi, theta, avar, n, igen, iseed, r)
```

```
var =
    0.7200
x =
     0
     0
     0
     0
     0
     0
     0
     0
     0
     0
iseedOut =
    11670341
     5471037
     7114334
    13114053
rOut =
    2.5000
    1.5000
    9.5000
         0
    0.4000
    0.2000
    1.0000
    0.4368
   -0.2149
ifail =
           0
```